Review of Basic Math Operations

LEARNING OBJECTIVES

1. Add, subtract, multiply and divide whole numbers.
2. Simplify fractions.
3. Find a least common denominator.
4. Add, subtract, multiply, and divide fractions and mixed numbers.
5. Convert fractions to decimal fractions.
6. Add, subtract, multiply, and divide decimal fractions.

OUTLINE

Introduction

There are four operations, or ways of performing a calculation, in mathematics: addition, subtraction, multiplication and division. No matter how complicated a calculation is, it will only involve some combination of these four operations. In this chapter you will have a chance to refresh your memory on the vocabulary and rules that apply to these math basics.

Addition and Subtraction

Two numbers that are added are called **addends**. The answer to an addition problem is called the sum or the **total**. In a word problem, when you see that the answer calls for a sum or total, it is a clue that addition is required for the solution.

Addend—A number that is added to another number.

Total—The answer to an addition problem.

EXAMPLE

Carl and Mary were in the pharmacy compounding the IV piggyback bags for the afternoon delivery. Mary made 44 piggyback bags and Carl made 27. How many IV piggybacks did they make in total?

SOLUTION

Mary made: 44 bags
Carl made: + 27
Total: 71

When two numbers are subtracted, the result is called the **difference**. When a word problem asks for an answer with more, less, or the difference between two numbers, it is an indication that subtraction will be used.

Difference—The answer to a subtraction problem.

EXAMPLE

How many more IV piggyback bags did Mary make than Carl?

SOLUTION

44 bags – 27 bags = 17 more bags

Multiplication and Division

Factor—That which is multiplied.

Product—The answer to a multiplication problem.

Numbers that are multiplied together are called **factor**s. The result of multiplication problem is called a **product**. As you will see later in this chapter, when performing mathematical operations on fractions, it is important to be able to find the factors that make up a given number.

EXAMPLE

Identify the factors in 21

SOLUTION

The factors are 3 and 7, because

$$3 \times 7 = 21$$

EXAMPLE

Find the product 8×7

SOLUTION

$8 \times 7 = 56$

Prime Number—A number (other than one) whose only factors are one and itself.

A number whose only factors are one and itself is called a **prime number**. The number one is not considered a prime number. Here is a list of the first several prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Quotient—The answer to a division problem.

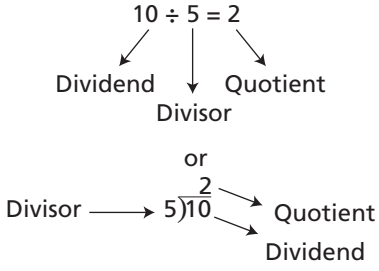
Divisor—A number by which another number is to be divided.

When numbers are divided, the result is called the **quotient**. The number being divided is called the dividend, and the number that is being used to divide is called the **divisor**. Division can be written several different ways.

EXAMPLE

How many ways can we write the problem 10 divided by 5?

SOLUTION



or

$$\text{Dividend} \longrightarrow \frac{10}{5} = 2 \longrightarrow \text{Quotient}$$

$$\downarrow$$

Divisor

When using a calculator to calculate a quotient, the dividend is always entered first.

EXAMPLE

Find the quotient.

a. $15 \div 3$ b. $\frac{20}{10}$ c. $9\overline{)63}$

SOLUTION

a. $15 \div 3 = 5$ b. $\frac{20}{10} = 2$ c. $9\overline{)\,7\;\;63}$

Sometimes the divisor does not go evenly into the dividend and we are left with some quantity left over, that is, a remainder.

EXAMPLE

Pharmacy technician Hugh Morris must repackage 465 tablets into bottles containing 20 tablets each. How many bottles are needed and how many tablets will be left over?

SOLUTION

$20\overline{)\,23\;\;465}$ Remainder 5

So, 23 bottles will be needed with 5 tablets left over.

Working with Fractions

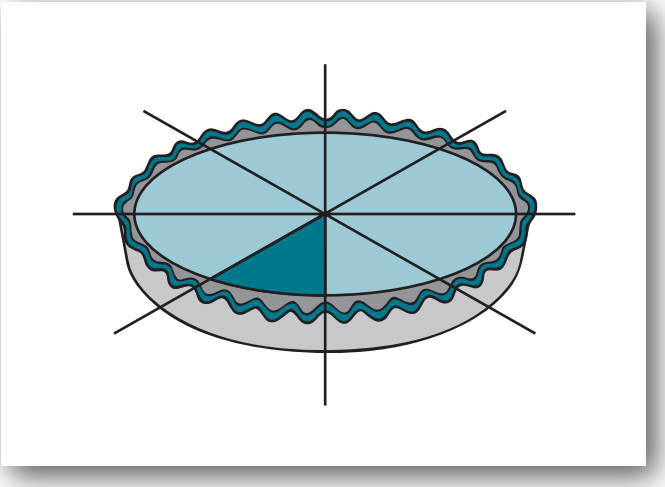
A fraction consists of a numerator (the top) and a denominator (the bottom) separated by a fraction bar. The numerator indicates the portions and the denominator indicates how many portions make a whole. For example, in the fraction ¾, the numerator "3" tells how many portions, and the denominator "4" indicates that four 4ths make a whole, or one unit. Figure 3-1 uses a pie to represent how a whole can be divided into equal portions.

The fraction bar is interpreted as a division symbol, so a fraction is really a kind of division problem. A fraction may also be interpreted as a ratio, or a relationship between two numbers.

TECH NOTE!

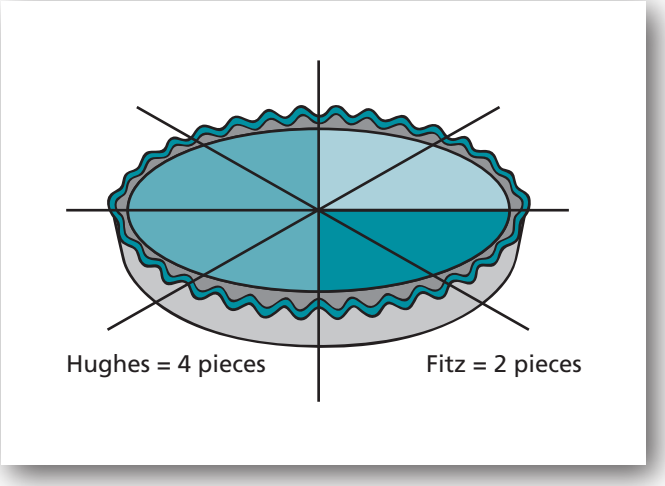
A ratio is the relationship between two quantities.

Figure 3-1. If a pie is cut into eight equal pieces, the whole pie could be described as 8 eighths, or 8/8. If one person eats a piece of pie (1/8 of the pie) there will be 7/8 left.



Look at the pie illustration in Figure 3-2. If two families share a pie, and one family eats two pieces while the other eats four, their pie consumption can be expressed as the ratio 2/4 or two to four. This is especially important in pharmacy, where compounding often requires you to think in terms of ratios, or parts per unit.

Figure 3-2. If the Fitz family eats two pieces of the pie, and the Hughes family eats four pieces, the ratio of their pie consumption can be expressed as 2/4 or two to four.



NUMBERS AT WORK

Understanding ratios or fractions is important for compounding medications and preparations.

To simplify a fraction, write the numerator and denominator as products of prime factors. Any factor that is the same in the numerator and in the denominator represents a factor of one and can be cancelled. Remaining factors are then multiplied to yield a simplified fraction.

EXAMPLE

Simplify $\frac{8}{50}$

SOLUTION

$$\frac{8}{50} = \frac{2 \times 2 \times 2}{2 \times 5 \times 5}$$

$$= \frac{2 \times 2}{5 \times 5}$$

$$= \frac{4}{25}$$

When multiplying fractions, the numerator of the first fraction is multiplied by the numerator of the second fraction and the denominator of the first fraction is multiplied by the denominator of the second fractions. Then the fraction is simplified. To simplify, first write the numerator and denominator as products of prime factors. Any factors that appear in both numerator and denominator can be cancelled before multiplying.

EXAMPLE

Each tablet contains 6/10 of a milligram of colchicine. How many milligrams of colchicine does one half a tablet contain?

SOLUTION

$$\frac{1}{2} \times \frac{6}{10}$$

$$= \frac{1 \times 2 \times 3}{2 \times 2 \times 5} = \frac{3}{2 \times 5}$$

$$= \frac{3}{10}$$

So one half of a tablet contains 3/10 of a milligram of colchicine.

The reciprocal of a fraction is obtained by inverting the original fraction, that is, by interchanging the **numerator** and **denominator**. For example, $\frac{4}{5}$ and $\frac{5}{4}$ are reciprocals. When dividing fractions, multiply the dividend by the reciprocal of the divisor.

Numerator— The portion of a fraction above the fraction line.

Denominator— The part of a fraction that is below the fraction bar, also the divisor of a division problem.

EXAMPLE

One half of a liter (metric measure of volume) of normal saline is to be divided into vials of 1/10 liter each. How many vials are needed?

SOLUTION

$$\frac{1}{2} \div \frac{1}{10} = \frac{1}{2} \times \frac{10}{1}$$

$$= \frac{2 \times 5}{2} = 5$$

So, five vials are needed.

To add (or subtract) fractions with the same denominator, add (or subtract) the numerators, place the sum (or difference) over the common denominator and simplify if possible.

EXAMPLE

One container holds 1/10 of a liter of concentrated sodium chloride, another contains 3/10 of a liter. The two solutions are to be combined. Would a ¼ liter or a ½ liter container be needed to hold the solution?

SOLUTION

$$\frac{1}{10} + \frac{3}{10}$$

$$= \frac{4}{10} = \frac{2 \times 2}{2 \times 5} = \frac{2}{5}$$

So, a 1/2 liter container is needed, because 2/5 liter is larger than 1/4 liter and less than 1/2 liter.

To add (or subtract) fractions with different denominators, a least common denominator must be calculated. The least common denominator can be calculated in two steps.

1. Write each denominator as the product of prime factors.
2. The least common denominator is the product of a repeated factor times any factors that are not repeated.

EXAMPLE

Find the least common denominator (lcd) of $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{9}{10}$.

SOLUTION

3 = 3
6 = 2 × 3 products of prime factors
10 = 2 × 5
2 and 3 are repeated factors, 5 is not.
So the lcd = 2 × 3 × 5 = 30

EXAMPLE

Find the lcd of $\frac{3}{14}$ and $\frac{1}{10}$

SOLUTION

$14 = 2 \times 7$
$10 = 2 \times 5$
2 is a repeated factor, 7 and 5 are not.
So the lcd = $2 \times 5 \times 7 = 70$

Once the least common denominator is calculated, each fraction is rewritten as an equivalent fraction that has the least common denominator. This is done by multiplying the numerator and denominator by the number that results in the common denominator.

$$\frac{3}{14} \times \frac{5}{5} = \frac{15}{70}$$
$$\frac{1}{10} \times \frac{7}{7} = \frac{7}{70}$$

EXAMPLE

The remains of three containers of Miralax® are to be combined to fill Mrs. Dennison's prescription. They are $\frac{1}{10}$ of a pint, $\frac{2}{5}$ of a pint and $\frac{1}{4}$ of a pint.

How many pints will be in the combined product?

SOLUTION

Find the lcd. $\frac{1}{10} + \frac{2}{5} + \frac{1}{4}$

$10 = 2 \times 5$
$5 = 5$
$4 = 2 \times 2$
One factor of 2 is repeated, one factor of 2 is not, and one factor of 5 is repeated
So the lcd = $2 \times 2 \times 5 = 20$

Now, we convert the denominators in each of the original fractions to 20. To accomplish this, multiply each fraction by a fraction equal to one (such as 2/2 or 5/5) so that the new denominator is 20.

$$\frac{1}{10} \times \frac{2}{2} + \frac{2}{5} \times \frac{4}{4} + \frac{1}{4} \times \frac{5}{5}$$
$$= \frac{2+8+5}{20}$$
$$= \frac{15}{20}$$
$$= \frac{3}{4}$$

So there is 3/4 of a pint in the finished product.

Fraction—A number usually expressed in the form a/b.

Improper Fraction—A fraction in which the numerator is greater than the denominator

A mixed number consists of a whole number and a **fraction**. For example $4\frac{3}{5}$ is a mixed number and is read, "four and three fifths." An **improper fraction** is one in which the numerator is greater than the denominator. To write a mixed number as an improper fraction, multiply the denominator by the whole number. Add this product to the numerator. Place this sum over the denominator.

EXAMPLE

Rewrite $2\frac{5}{7}$ as an improper fraction.

SOLUTION

$7 \times 2 = 14$ multiply denominator by whole number

$14 + 5 = 19$ add product to numerator

$\frac{19}{7}$ place sum over denominator

To write an improper fraction as a mixed number, divide numerator by denominator, and place the remainder over the denominator.

EXAMPLE

Rewrite $\frac{25}{3}$ as a mixed number.

SOLUTION

$$3\overline{)25}\quad 8$$
$$\underline{-24}$$
1 remainder

So $\frac{25}{3} = 8\frac{1}{3}$

To multiply or divide two mixed numbers, write each mixed number as an improper fraction, then multiply or divide the resulting fractions.

EXAMPLE

Tylenol® with codeine solution is to be administered in doses of 2 ½ teaspoonfuls six times per day if needed for pain. How many teaspoonfuls of Tylenol® with codeine may be given in a day?

SOLUTION

$$2\frac{1}{2} \times 6 = \frac{5}{2} \times \frac{6}{1}$$

$$= \frac{30}{2}$$

The result can then be written as a whole number.

$\frac{30}{2} = 15$ teaspoonfuls

So 15 teaspoonfuls of Tylenol® with codeine may be given in a day.

EXAMPLE

Find the quotient.

$$3\frac{1}{2} \div 2\frac{1}{3}$$

SOLUTION

$$3\frac{1}{2} \div 2\frac{1}{3}$$
$$= \frac{7}{2} \div \frac{7}{3}$$
$$= \frac{7}{2} \times \frac{3}{7}$$
$$= \frac{3}{2}$$
$$= 1\frac{1}{2}$$

To add or subtract mixed numbers, write each as an improper fraction, find the least common denominator, then add or subtract the resulting fractions. The answer can then be written as a mixed number.

EXAMPLE

Dr. Lance Boyle prescribes 2½ teaspoonful of docusate liquid and 1¼ teaspoonful of Metamucil®. How many teaspoonfuls of medicine are prescribed?

SOLUTION

$$2\frac{1}{2} + 1\frac{1}{4} = \frac{5}{2} + \frac{5}{4} \quad \text{change to improper fractions}$$
$$= \frac{5}{2} \times \frac{2}{2} + \frac{5}{4} \quad \text{find lcd}$$
$$= \frac{10}{4} + \frac{5}{4}$$
$$= \frac{15}{4} = 3\frac{3}{4} \quad \text{change to mixed number}$$

So 3¾ teaspoonfuls of medicine are prescribed.

Decimal fractions

A **decimal fraction** is a fraction in which the denominator is a power of 10 such as 10, 100, or 1000. The denominator of a decimal fraction is not written, but the number of digits to the right of the decimal indicates the place value of the denominator. One digit indicates tenths, two indicates hundredths, three indicates thousandths, four indicates ten thousandths, and so on (see Figure 2-3, in Chapter 2). Any fraction can be written as a decimal fraction by dividing the numerator by the denominator. When using a calculator, make sure to enter the numerator first.

Decimal Fraction—A fraction whose denominator is a power of ten.

EXAMPLE

Write $\frac{1}{8}$ as a decimal fraction.

SOLUTION

$$
\begin{array}{r}
0.125 \\
8\overline{)1.000} \\
\underline{-\ 8} \\
20 \\
\underline{-16} \\
40 \\
\underline{-40} \\
0
\end{array}
$$

Order of Operations

Mathematical problems are often written out in the form of an equation. An **equation** is a mathematical statement, separated by an equal sign, where each side of the statement is equal. Take a look at the example of an addition problem at the beginning of the chapter, where the two technicians were making IV piggyback bags. When that problem is written as an equation it looks like this:

Equation—A mathematical statement of equality

$$44 \text{ bags} + 27 \text{ bags} = 71 \text{ bags}$$

We can see that 44 + 27 is another way of expressing the number 71. Thus, each side of an equation is referred to as an expression. All word problems can be written as equations, but sometimes these equations will contain more than one mathematical operation, so it is important to know in what order to perform them.

The rule is that any operation isolated by parentheses is completed first, followed by multiplication and division from the left of the problem to the right and then addition and subtraction from the left to the right.

TECH NOTE!

Perform math operations in parentheses first, then multiplication and division, from left to right, then addition and subtraction from left to right.

EXAMPLE

Theresa Jones, R.Ph., is the pharmacist-in-charge for Just Right Pharmacy. She is looking at the cost of buying pharmaceuticals for the past quarter, and wants to determine the average cost per month. Pharmaceuticals cost $19,052.00 in January, $14,363.00 in February, and $22,103.00 in March. What was the average cost/month of pharmaceuticals for the quarter?

SOLUTION

Remember that to find an average of a series of numbers, you find the sum of the numbers and then divide by the number of entries in the series.

$$\frac{(\$19{,}052.00 + \$14{,}363.00 + \$22{,}103.00)}{3 \text{ months}} = \$18{,}506/\text{month}$$

Continue to practice and refresh the math skills reviewed in this chapter by completing the practice problems since these skills serve as the foundation for upcoming chapters.

Practice Problems

1. Find the sum.

a. $3 + 9$

b. $127 + 13$

c. On Monday, Bill the pharmacist sees 19 customers for prescription consultations and pharmacist Don sees 18 customers for consultations. Altogether, how many customers were seen for consultations on Monday?

2. Find the difference.

a. $18 - 7$

b. $124 - 39$

c. The C. F. Eye Care hospital had 100 bottles of artificial tears eye drops on the shelf. 59 bottles were removed because the expiration date had passed. How many bottles remained on the shelf?

3. Find the product.

a. 8×7

b. $3 \times 4 \times 14$

c. Dr. Dee Kay orders that 2 capsules of Zovirax® 200 mg be administered 5 times daily for 14 days. How many capsules are needed?

4. Find the quotient.

a. $100 \div 200$ b. $2\overline{)18}$ c. $\frac{14}{6}$

5. Three hundred vitamin C 250 mg tablets are to be equally distributed among 150 patients.

a. How many tablets will each patient receive? Will there be any tablets left over? If so, how many?

b. One thousand tablets are to be divided into prescription vials containing 30 tablets each. How many vials are needed? Will there be any tablets left over? If so, how many?

6. Simplify each fraction.

a. $\frac{3}{12}$ b. $\frac{14}{42}$ c. $\frac{8}{18}$

7. Find the product. Write each product in simplest form.

a. $\frac{2}{9} \times \frac{3}{4}$ b. $\frac{9}{28} \times \frac{14}{27}$ c. $\frac{3}{10} \times \frac{5}{6}$

8. Find the quotient. Write each quotient in simplest form.

a. $\frac{2}{9} \div \frac{4}{3}$ b. $\frac{9}{10} \div \frac{6}{5}$

9. Find the quotient. Write each quotient in simplest form.

a. $\frac{2}{27} \div \frac{4}{9}$

b. $\frac{3}{4}$ of an ounce of hydrocortisone 1% is to be divided into three equal parts. How much will each part contain?

10. Find the sum or difference. Write each in simplest form.

a. $\frac{3}{8}+\frac{2}{8}$ b. $\frac{3}{4}-\frac{1}{4}$ c. $\frac{1}{9}+\frac{2}{9}$ d. $\frac{7}{8}-\frac{4}{8}$ e. $\frac{7}{10}-\frac{3}{10}$

11. Find the sum or difference. Write each in simplest form

a. $\frac{2}{3}+\frac{1}{4}$ b. $\frac{3}{4}+\frac{1}{8}$ c. $\frac{2}{3}-\frac{1}{12}$ d. $\frac{2}{9}+\frac{5}{12}$ e. $\frac{8}{9}-\frac{2}{5}$

12. Rewrite the mixed number as an improper fraction.

a. $2\frac{3}{4}$ b. $9\frac{1}{8}$ c. $4\frac{2}{3}$

13. Rewrite the improper fraction as a mixed number.

a. $\frac{14}{3}$ b. $\frac{21}{5}$ c. $\frac{14}{9}$

14. Find the product.

a. $2\frac{1}{3}\times\frac{3}{4}$ b. $4\frac{1}{2}\times 1\frac{1}{9}$ c. $2\frac{2}{3}\times 1\frac{1}{4}$

15. Find the quotient.

a. $6\frac{2}{3}\div\frac{5}{3}$

b. 3 1/2ounces of Robitussin DM® is to be divided into portions that are 1/4 ounce each. How many portions will there be?

c. 2 1/2 tsp of amoxicillin suspension is to be given in two equal doses. How many tsp will there be in each dose?

16. Find the sum.

a. $3\frac{1}{6}+1\frac{5}{8}$

b. Baby George is to be given 2 1/2 teaspoonfuls of prednisone 1 mg/mL each morning and 1 1/4 teaspoonfuls in the afternoon. How many teaspoonfuls of prednisone are to be taken each day?

17. Rewrite the fraction as a decimal fraction.

a. $\frac{3}{8}$ b. $\frac{9}{10}$ c. $\frac{4}{5}$

18. Dr. Denton prescribes one tablet of pseudoephedrine 60 mg to be taken twice daily for one month, for patient Constance Noring. How many tablets should be dispensed?

19. Dr. Gohan N. Sumi directs patient Nora Maki to take one capsule of Augmentin® 250 mg three times a day for 14 days. How many capsules should be dispensed?

UNIT 1

20. Mrs. Johnson is asked to give her daughter Pam ¾ teaspoonful of Bactrim® Suspension 2 times a day for 14 days. The pharmacist fills the order with a bottle of 20 tsp. Is this enough to last for 30 days?

21. Wanda Hu gets three prescriptions filled every month. Although her insurance company provides prescription coverage, she pays a co-pay for each prescription. For her birth control tablets she pays $15.00, for her albuterol inhaler she pays $15.00, but for her Advair® inhaler she pays $65.00 each month. What is the average co-pay Wanda pays?

22. At the class picnic hot dog eating contest, the senior class representative eats 12 hot dogs in 15 minutes, while the junior class contestant can only manage 8 hot dogs in 15 minutes.

a. Write each class's hot dog eating results as a ratio of hot dogs/time, and reduce each to hot dogs per minute.

b. Which class won the contest?

23. Nat Faste, the representative from Code Blue Insurance Company, has granted approval for Wright Pharmacy to fill a 90-day supply of Evan Tooly's prescription. He takes two ibuprofen 400 mg tablets three times daily. How many tablets does Nat need to fill this prescription?

24. Complete the table.

Fraction	Decimal	Percent
1/2	0.5	50%
	0.375	37.5%
3/4		75%
	0.80	80%
	0.25	25%
2/5		40%

25. The pharmacist asks the pharmacy technician to divide 2,000 grams of zinc oxide ointment into several sized jars. He would like the technician to fill ten 60-gram jars, eleven 90-gram jars and six 30-gram jars.

a. What is the total amount of zinc oxide used to fill all the jars.

b. Write the ratio of the amount in the 60 gram jars over the total amount of zinc oxide ointment used and reduce to the simplest form.

c. Write the fraction determined in b. as a decimal.